# Study on the Positive Solution of Integral Boundary Value Range of Fractional Differential Equation

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**Abstract:** In the past few decades, the research on fractional differential theory has developed rapidly all over the world. More and more fractional differential equations are used to describe the problems related to mathematical anomalous diffusion, high energy physics, system control, optical and thermal system research, bioengineering and other application fields. The relevant literature around this key academic content is also very much. In this paper, we hope to make a deep research and use the legette Williams fixed point theorem, which is widely used at present, to analyze the solution method of integral positive solution of fractional differential equation with integral boundary value range basic condition and the existence result problem of positive solution, and to verify the validity of the result with the case calculation process.

## 1. Introduction

At present, fractional differential equations have been used in most research fields, such as control engineering, biomathematics, hydrodynamics and so on. In order to prove the existence of fractional differential equations, many scholars have made a lot of research, given their own opinions, and obtained a lot of research results. There are a lot of research results and literature also indicate that the existence of positive solution of fractional differential equation under the condition of integral boundary value range exists, which is worthy of further study.

## 2. On the Existence of Positive Solutions to the Boundary Value Range Problems of Fractional Differential Equations

At present, there are many literature studies that show the existence of positive solutions for a class of boundary value range related problems of fractional differential equations with integral boundary conditions. It makes use of the classical Guo KRAS noselskii fixed point theorem and obtains the existence of at least one positive solution in the problem. In addition, some scholars have also studied the characteristics of the integral boundary value range of singular nonlinear conjugate fractional differential equations with nonlocal terms in fractional differential equations, and pointed out the sufficient conditions for the existence of positive solutions. In fact, there are many scholars who use monotone iterative method or Green's function inequality to solve work nonlinear fractional boundary value problems. They have proved the existence of nontrivial solutions in this kind of equations. Of course, what we hope to use in this paper is Leggett Williams

fixed point theorem. First, we hope to obtain the existence solutions of multiple positive solutions in fractional differential equations. Then, we use Guo Krasnoselskii fixed point theorem and upper and lower solution methods to solve the existence and even uniqueness of the positive solutions of this problem. At least three positive solutions are obtained in the process of solving, in order to prove the benign use of this method The effect is [1]. It is also proved that the solution of boundary value problem of fractional differential equation is effective and necessary

From the analysis results of the existing research, there are many cases that study the results of fractional differential equations with integral boundary conditions. Most of them have established the non-linear term without derivative term, and the concept of first derivative term has been incorporated into the research process. In the following, I hope to give a real way to deal with this problem through a real topic case, so as to realize the effective promotion of the research content and make it more necessary and universal [2].

## 3. Study on the Existence of Positive Solutions to the Integral Boundary Value Range Problems of Fractional Differential Equations

In this paper, we want to study the existence of positive solutions to the boundary value range problem of fractional differential equation, so we should first put forward its Research Preparatory knowledge, and give the basic calculation form of the boundary value problem of fractional differential equation as follows [3]:

$$
\begin{aligned}
& D_0^q + u(t) + f\left(t, u(t), u'(t)\right) = 0 \left(0 < t < 1, 3 < q < 4\right) \\
& u(0) = u(1) = 0 \\
& a_1 u'' - \beta_1 u'' = \int_0^1 h(t) u''(t) \, dt, \gamma_1 u'''(0) - \delta_1 u'''(1) = \int_0^1 g(t) u'''(t) \, dt
\end{aligned}
\tag{1}
$$

In the above calculation form, we can use the important fixed point theorem that we hope to generalize in this paper - Leggett Williams fixed point theorem to solve the boundary value problem from (0.1) ~ (0.3) to three solutions, which are $u_1(t)$、 $u_2(t)$、 $u_3(t)$ respectively. Finally, we give a specific numerical example and verify the correctness of the results. It is also proved that the solution of boundary value problem of fractional differential equation is effective and necessary [4].

## 4. Study on the Existence of Positive Solutions for the Integral Boundary Value Range of Fractional Differential Equations

### 4.1. Solving Process

By using the Leggett Williams fixed point theorem, which is being widely extended up to now, it is assumed that there is a Banach space in a point, and there is $P \subset E$ as the cone of $E$ in the space. In the calculation, if the given constant is $r_2 \geq d > b > r_1 > 0$, $L_2 \geq L_1 > 0$, it can be assumed that the attribute of $\alpha$、 $\beta$ value in the given function is concave functional and $\gamma$ is convex

functional, then they meet the basic calculation morphological conditions for solving the boundary value problem of fractional differential equation, and all operators can be regarded as a complete and completely continuous operator, of course, it also meets the following two conditions [5]:

$\alpha(Tu) < r_1, \beta(Tu) < L_1$, Put forward $u \in \overline{P}(\alpha, r_1, \beta, L_1)$

$\gamma(Tu) > b$, Put forward $u \in \overline{P}(\alpha, d, \beta, L_2, \gamma, b), \alpha(Tu) > d$

In this case, the three fixed points of the operator $T$ in $\overline{P}(\alpha, r_2, \beta, L_2)$ are respectively $u_1$、$u_2$、$u_3$. In this stage, the differential equation theory can be introduced to get the following results:

$$
\begin{aligned}
u''(t) + y(t) &= 0, 0 < t < 1 \\
u(0) &= u(1) = 0
\end{aligned}
\tag{2}
$$

From the above formula, the solution form is $u(t) = \int_0^1 G(t,s) y(s) ds$, among which are:

$$
G(t,s) = \begin{cases} t(1-s), 0 \le t \le s \le 1 \\ s(1-t), 0 \le s \le t \le 1 \end{cases}
\tag{3}
$$

The above solvable form can prove the $G(t,s)$ correlation and always assume the following conditions, namely:

$$
M(s) = \frac{(1-s)^{q-4}[(1-s)(\gamma_1 - \delta_1)\beta_1 + (q-3)\beta_1 + (q-3)\beta_1\delta_1]}{(\alpha_1 - \beta_1)(\gamma_1 - \delta_1)\Gamma(q-2)}
\tag{4}
$$

From the above solution, we can find the corresponding linear equation content in the solution of boundary value problem of fractional order differential equation [6].

## 4.2. Result Analysis

In the process of solving the boundary value of fractional differential equation, at least (0.1) ~ (0.3) three solutions are obtained by using the legette Williams fixed point theorem, and the corresponding linear equation content in the solution is clarified. In the whole solution process, the universal function $a(u)$ is also defined for any $u \in P$, as follows [7]:

$$
a(u) = \max|u(t)|, \beta(u) = \max|u'(t)|, \gamma(u) = \min|u(t)|
\tag{5}
$$

In the above formula, $a$、 $\beta$、 $\gamma$ can satisfy all the conditions of the above solution process, and there is $a$、 $\beta$、 $\gamma$. Combined with this, the definition operator is given as follows [8]:

$$
(Tu)(t) = \int_0^1 G(t, \tau) \int_0^1 H(\tau, s) f(s, u(s), u'(s)) ds d\tau
\tag{6}
$$

In the above definition of operators, we should combine the continuity characteristics of the operators, directly verify $T$, establish $P - P$'s full continuity operators, and analyze the existence of three solutions (0.1) ~ (0.3) in the boundary value problem, and finally obtain the existence characteristics of fixed points equivalent to the operator $T$、 $P$, so as to facilitate the effective

writing of mathematics [9].

## 5. On the Existence of Positive Solutions to the Integral Boundary Value Range Problems of Fractional Differential Equations

In the fractional differential equation, the problem of its integral boundary value range needs to be considered most. As has been confirmed in the above, the existence of positive solutions to its related problems must be optimized and calculated in place. In particular, it needs to be further analyzed in combination with solving cases. For example, in the boundary value problem, when

$$f(t,w,z)= \frac{1}{2(1+t)^2}\left(1+te^{tw-2}+z\right), g(s)=2, h(s)=4s^{\frac{3}{2}}, \text{ suppose the specific example is } \alpha =\frac{3}{2} \text{ [10].}$$

First of all, it is necessary to prove the upper and lower solutions of the boundary value problem. Considering all the conditions in the proof theorem, it can meet the requirements. Then, it is necessary to further solve the maximum lower solution value $\bar{x}$ and the minimum upper solution value $\bar{y}$ in combination with the boundary value problem in the theorem, and meet the condition $0\le t^{0.75} \le x(t)\le \bar{y}(t)\le t^{-1.02}$.

As can be seen from the above case study, the research on fractional differential equations in academic circles has been quite in-depth, for example, the application in science and engineering has become more and more extensive. The research and development of fractional differential theory is quite rapid in the world. More and more fractional differential equations are used to describe the problems related to mathematical anomalous diffusion, high-energy physics, system control, optical and thermal system research, bioengineering and other application fields. There are also a lot of relevant literature around this key academic content, from the existing relevant research According to the analysis results, there are many cases that study the results of fractional differential equations with integral boundary conditions. Most of them have established the nonlinear term without derivative term, and the concept of first derivative term has been incorporated into the research process. In the following, I hope to give a real way to deal with this problem through a real topic case, so as to realize the effective promotion of the research content and make it more necessary and universal [2].

In order to prove the existence of fractional differential equations, many scholars have made a lot of research, given their own opinions, and obtained a lot of research results. There are a large number of research results and literature also indicate that the existence of positive solution of fractional differential equation under the condition of integral boundary value range exists, which is worthy of further study. Even some scholars at home and abroad have made multi angle analysis on the boundary value problem of fractional differential equation in the process of using nonlinear analysis method, in order to prove it from different angles The existence of the boundary value is

shown in order to obtain good calculation results [11]. In addition to the calculation and solution method of Leggett Williams fixed point theorem mentioned in this paper, the algorithm of fixed point theorem adopted by avery Peterson is quite mature, which can realize scientific and reasonable solution of boundary value problem of fractional order differential equation, and can provide sufficient calculation conditions of at least three positive solutions, allow various singular phenomena to appear in the calculation process, and clarify the fraction Second derivative problem. In fact, there are many scholars who use monotone iterative method or Green's function inequality to solve work nonlinear fractional boundary value problems. They have proved the existence of nontrivial solutions in this kind of equations. Of course, what we hope to use in this paper is Leggett Williams fixed point theorem. First, we hope to obtain the existence solutions of multiple positive solutions in fractional differential equations. Then, we use Guo Krasnoselskii fixed point theorem and upper and lower solution methods to solve the existence and even uniqueness of the positive solutions of this problem. At least three positive solutions are obtained in the process of solving, in order to prove the benign use of this method Effect.

## 6. Conclusion

At present, the fractional differential equation has been widely used in engineering and other fields, and the research on the fractional differential equation with integral boundary conditions is also very rich. In the process of research, it is necessary to understand the content of the first derivative item in the nonlinear item reasonably, and give different methods to deal with different problems, so as to ensure the fresh connotation of knowledge application in the teaching process. For example, in this paper, in the process of solving the integral boundary value range of fractional differential equation using Leggett Williams fixed point theorem, the three-point boundary value problem and even the multi-point boundary value problem are analyzed. The existence and uniqueness of the positive solution of the problem are solved by using Guo krasnoselski fixed point theorem and the upper and lower solution methods, and good research results are obtained through the above research.. In order to prove the existence of fractional differential equations, many scholars have made a lot of research, given their own opinions, and obtained a lot of research results. There are a lot of research results and literature also pointed out that the existence of positive solutions of fractional differential equations under the condition of integral boundary value range exists, which is worthy of further study. These research results are quite valuable and worthy of further study and promotion in the future. Therefore, in general, this paper hopes to take many professional research contents as the basis, in order to obtain better research results and achieve more technological breakthroughs [12].

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